SOME LIMITS ON THE PERFORMANCE OF AN ANALOG OPTICAL LINK

C. H. Cox III, E. I. Ackerman, and G. E. Betts

Lincoln Laboratory, Massachusetts Institute of Technology
Lexington, MA 02173-9108

Phone: (617) 981-1036 FAX: (617) 981-3905 email: ccox@ll.mit.edu

ABSTRACT

Recent performance improvements in analog optical links have prompted us to analytically investigate the limits to link performance. In this paper these limits are derived and compared to current state-of-the-art link performance.

INTRODUCTION

In many applications of analog fiber optic links, the link noise figure (*NF*) and intermodulation-free dynamic range (*IM-free DR*) are two of the most important parameters. Ideally it would be most beneficial to improve both these parameters simultaneously. However many linearization techniques investigated to date, especially broadband ones, increase the *IM-free DR* at the expense of an increased *NF* [1]. Conversely, attempts to reduce the noise figure below its limits will needlessly sacrifice *IM-free DR*. Thus it is important to understand the limits on the improvements that can be made in *NF* and *IM-free DR*. In this paper some of these limits on *IM-free DR* and *NF* will be presented and explained. The scope of the discussion will be on the link topology universally in use today: the intensity-modulation/direct-detection topology.

MAXIMUM SNR IN RIN-LIMITED LINKS

Although it is often assumed, usually to facilitate the analysis, that a link is shot-noise limited, in fact many practical links (virtually all direct modulation links among them) are laser relative intensity noise (*RIN*) limited. For intensity modulation links, the residual intensity fluctuations at the photodetector, with no signal applied to the modulation device, clearly will limit the minimum modulation signal that can be conveyed by the link. Consequently it is clear intuitively that the *RIN* must affect the limit on signal-to-noise ratio.

To formalize this limit, consider the photodetector shown in Figure 1. Since *RIN*-limited detection is being assumed here, that is the only noise source shown. The magnitude S of the signal modulating the detected light is related to the average photodetector current I_D by the optical modulation depth *OMD*, *viz.*:

$$S = \frac{OMD\, I_D}{\eta_D}, \tag{1}$$

where η_D is the photodetector responsivity (which has units of A/W).

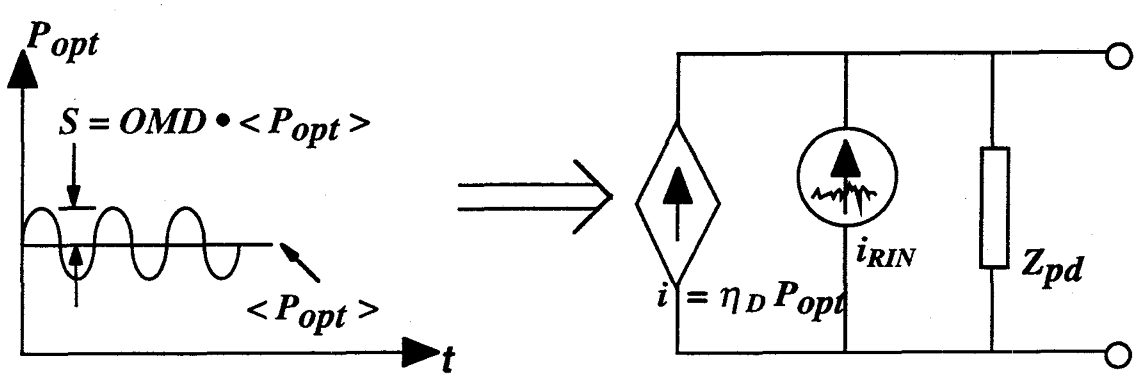

Fig. 1. Schematic diagram of the photodetector in an analog optical link. *RIN*-limited detection is assumed.

Under the assumption of *RIN*-limited detection, the magnitude N of the noise present on the detected optical carrier is:

$$N = \frac{I_D \sqrt{RIN\ B}}{\eta_D}, \tag{2}$$

where B is the instantaneous bandwidth. Therefore, the link's output signal-to-noise ratio *SNR* can be written as the square of the ratio of S to N (the squaring is necessary because the detector converts optical power to electrical current); *i.e.*:

$$SNR \equiv \left(\frac{S}{N}\right)^2 = \frac{OMD^2}{RIN\ B}. \tag{3}$$

However recall that one definition of *RIN* is

$$RIN \equiv \frac{\langle i_D^2 \rangle}{\langle i_D \rangle^2} = \frac{\langle i_D^2 \rangle}{I_D^2}. \tag{4}$$

Substituting equation (4) into (3) yields:

$$SNR = \frac{OMD^2\ I_D^2}{\langle i_D^2 \rangle\ B}. \tag{5}$$

The maximum *OMD* is 1, so the maximum *SNR* under *RIN*-limited detection is simply

$$SNR_{\max} = \frac{I_D^2}{\langle i_D^2 \rangle\ B} = \frac{1}{RIN\ B}. \tag{6}$$

As a practical example, consider the CATV application, for which typically $OMD = 4\%$. Distributed-feedback (DFB) lasers, which usually have a *RIN* around –155 dB/Hz, are often used for this application. This combination of parameters results in a maximum CNR = 61 dB for $B = 4$ MHz, which is also representative of CATV applications [2]. This analysis makes clear that for *RIN*-limited detection the only two ways to improve the *SNR* are either to reduce the *RIN* or to increase the *OMD*. The CATV industry has focused primarily on increasing the *SNR* by reducing the *RIN*.

Over the last few years there has been considerable interest in increasing the linearity of analog optical links, which in turn would permit increasing the *OMD*. A variety of linearization approaches has been investigated both theoretically and experimentally. Improvements of 10 to 20 dB in the *IM-free DR* have been demonstrated, which are sufficient to meet the needs of the CATV application [1, 2]. A natural question is: how much more improvement is possible with better linearization schemes? Perfect linearization would result in a linear modulation function, independent of the modulation signal amplitude. Recall that the *IM-free DR* is defined as the highest *SNR* for which the intermodulation terms are equal to the noise floor. But with perfect linearization, there are no intermodulation terms, so the maximum *IM-free DR* is simply $SNR_{\max}$. Although linearization was originally motivated by a need to reduce distortion, as was stated above, linearization also has the additional advantage of permitting a larger *OMD* to be used for a given amount of distortion, thereby enabling the *SNR* to more nearly approach the *RIN*-limited value.

To use the above limits as a gauge against which to judge current linearization methods, consider the reported results for external modulation links. Without linearization, an external modulation link using a standard Mach-Zehnder modulator has an *IM-free DR* of about 110 dB•Hz$^{2/3}$ and a maximum *SNR* of 160 dB•Hz [3]. Using a solid state laser, the *RIN* is not directly measurable, but measurements from which *RIN* can be inferred place an upper bound of –180 dB/Hz. The best reported linearization methods have *IM-free DR*'s on the order of 135 dB•Hz$^{4/5}$ when used with the same optical power as the link with a standard modulator [4]. Thus there is room for an improvement in *IM-free DR* of about 45 dB relative to what has been demonstrated to date.

NOISE FIGURE AND GAIN

In an earlier paper [5] the limits on noise figure and their relationship to link gain were presented; they are summarized here. It can be shown that the limiting value of amplifierless link *NF* is related to the link gain G by the following simple relationship:

$$NF_{\min} = 10\ \log\left[2 + \frac{1}{G}\right]. \tag{7}$$

There are two assumptions that are necessary to apply this limit. One assumption is that the link input is passively matched to the source resistance. The other assumption used in deriving the above limit is that the passive matching network is lossless—this restriction will be removed in the discussion below. The limit can be decomposed into two parts: a gain-independent *passive match limit*, and a gain-dependent *passive attenuation limit*. The *passive match limit* of 3 dB, which corresponds to the factor of 2 in the above expression, is due to the real—*i.e.*, thermal—resistance of the modulation device at the input of the link; hence it is independent of the link gain. The *passive attenuation limit* of $1/G$ is due to the thermal noise at the link output.

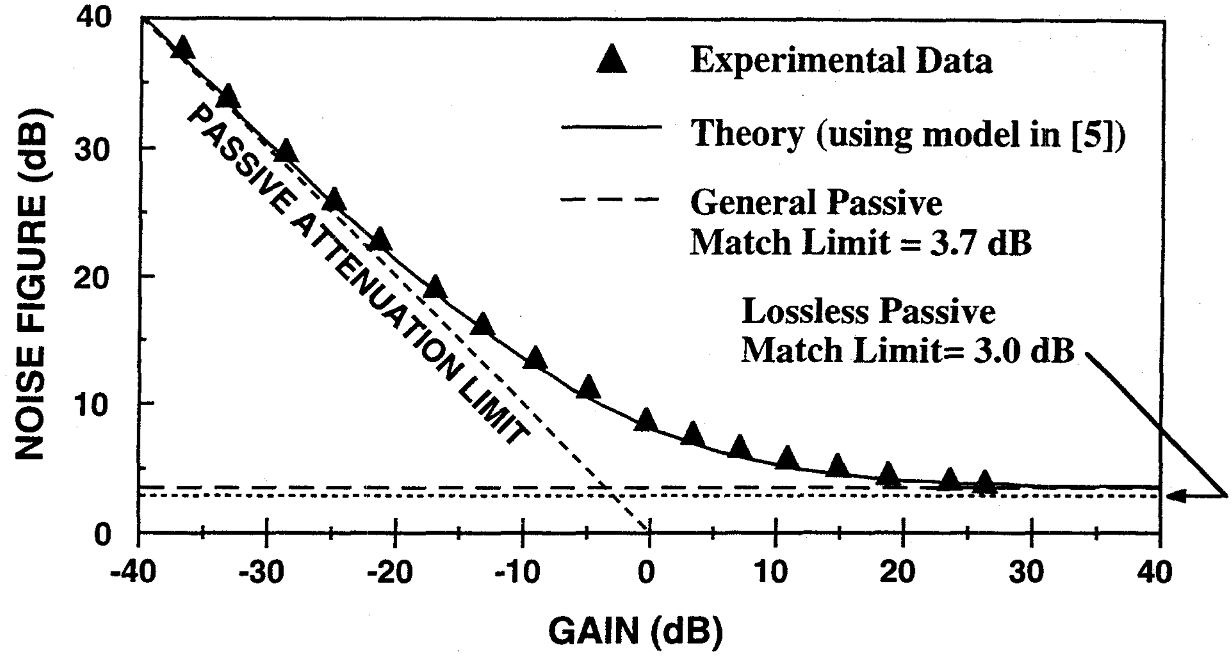

Fig. 2. Relationship between gain and noise figure for an experimental external modulation link with input matching circuit loss of 0.7 dB [after 5].

This output noise, which is independent of G, when referred to the link input is divided by G—hence the $1/G$ dependence of this term. The name for this limit arises from its analogy to the relationship between the noise figure and attenuation of a passive attenuator. However there is a subtle difference between a passive attenuator and an optical link. Whereas the noise figure of a 3 dB attenuator is equal to 3 dB, the noise figure for a link with 3 dB of RF-to-RF loss (or $G = 0.5$) is at best 6 dB (= 10 log [2 + 1/0.5]).

In practice any realizable circuit has loss, so this needs to be taken into account when applying the limit in actual situations. It can be shown that the matching circuit with loss can be represented as a lossless matching circuit preceded by an attenuator whose attenuation equals the matching circuit loss. Thus this loss simply adds a constant term to the lossless limit. This new limiting value, which equals 3 dB plus the matching circuit loss in dB, can only be achieved when the laser *RIN* is negligible in comparison to the detector shot noise.

Recently measurements of gain and noise figure were made on an external modulation link which met the above conditions. These measured link data are plotted in Figure 2. In general the data agree well with the limits presented here. The input matching circuit loss was measured independently to be 0.7 dB, yielding a noise figure limit (for $G \rightarrow \infty$) of 3.7 dB. At 26.5 dB of link gain, the noise figure was measured to be 4.2 dB.

Although it is commonplace to plot the link gain and noise figure as a function of link average optical power, it is also instructive to plot the link noise figure and dynamic range as a function of modulator sensitivity. In the case of a Mach-Zehnder modulator, V_π is the appropriate measure of modulator sensitivity (where smaller V_π corresponds to greater sensitivity). Figure 3 shows as a function of V_π the analytically determined noise figure and third-order intermodulation-free dynamic range of a losslessly impedance-matched external modulation link that uses a standard Mach-Zehnder intensity modulator.

The link's intermodulation-free dynamic range is a ratio whose denominator is proportional to *NF*. The numerator of this ratio is determined by intermodulation distortion and as such has a consistent inverse-squared dependence on V_π. Region I of the plot in Figure 3 corresponds to the passive attenuation limit case, for which the link's *NF* is approximately equal to $1/G$ and is therefore proportional to $(V_\pi)^2$. Thus *IM-free DR* is invariant with respect to V_π for this range of V_π's. Conversely, in Region III, *NF* has nearly reached the passive match limit of 3 dB, and is unaffected by further reductions in V_π. Therefore further reductions in V_π cause a decrease in *IM-free DR*.

Note from Figure 3 that the left-hand extreme of Region II has roughly the same *IM-free DR* but lower *NF* than any operating point in Region I. Moreover, the right-hand extreme of Region II has roughly the same *NF* but greater *IM-free DR* than any point in Region III. Region II therefore represents the best trade-off between noise figure and dynamic range. The values of V_π at the boundaries of Region II depend on the laser's CW optical power output and other link parameters.

SUMMARY

It has been shown that recently reported improvements in optical analog link performance—achieved via external modulator linearization and/or the noise figure reduction resulting from using large optical power and off-biasing the modulator—do not yet approach the upper limits we have derived. Perfect linearization of the modulator transfer function could yield an intermodulation-free dynamic range comparable to the maximum achievable signal-to-noise ratio, which is physically limited only by the RIN of the laser.

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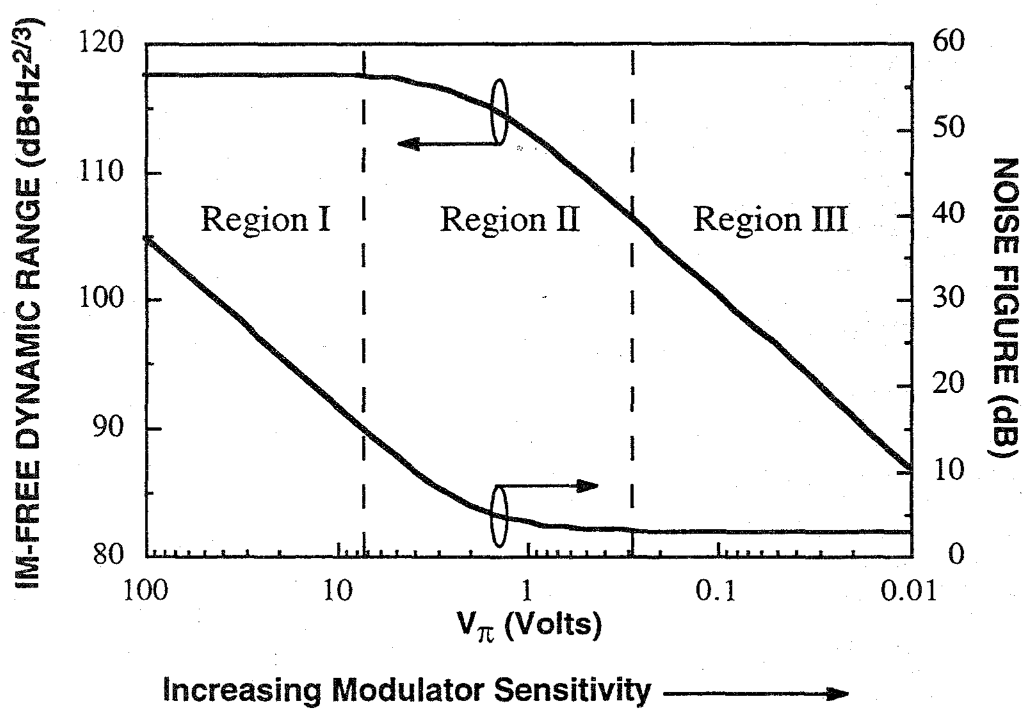

Fig. 3. Relationship between modulator sensitivity and external modulation link performance. Noise figure and third-order intermodulation-free dynamic range are shown as functions of the modulator V_π.