# Gain Limit in Analog Links Using Electroabsorption Modulators

G. E. Betts, X. B. Xie, I. Shubin, W. S. C. Chang, and P. K. L. Yu

*Abstract*—Analog optical links using electroabsorption modulators have a gain limit caused by the photocurrent in the modulator. The gain limit results in a minimum link noise figure as well. The gain limit is due to the voltage-dependent absorption causing a voltage-dependent component to the photocurrent; the gain limit applies to a modulator that has this relationship.

*Index Terms*—Electroabsorption, gain, high power, modulator, optical analog link.

#### I. INTRODUCTION

**I** N AN external modulation analog link, it is routine to think of the modulator as an ideal three-terminal device where the light is controlled by the voltage applied to the modulator, but there is no effect of the light on the voltage. This picture is appropriate for modulators where the modulation is based on the linear electrooptic effect. For direct modulation links, the light is produced by the current supplied to the transmitter laser, so there is a direct relation between electrical power supplied to the optical transmitter and the light output. This results in a limitation on the gain of direct modulation links that does not exist for external modulation links [1].

Electroabsorption modulators are intermediate between these two pictures. They are external modulators and they do affect the light through voltage-controlled absorption. However, the absorption produces photocurrent, which interacts with the electrical circuit. At low optical power, the electroabsorption modulator behaves like an ideal external modulator, but at high optical power it exhibits a gain limit.

This effect of photocurrent on gain was noticed when electroabsorption modulators began to be able to handle optical powers of several milliwatts [2]. This led to the observation that there was a limit on the modulation efficiency of the electroabsorption modulator as the optical power increased [3]. Here we show how this limit arises and how it limits the performance of analog links using electroabsorption modulators. We also show experimental data confirming the link gain limit at very high optical power levels.

# II. THEORY

The basis for our analysis is the equivalent circuit shown in Fig. 1. We represent the photocurrent effect by a resistor because

Manuscript received May 18, 2006; revised July 3, 2006. This work was supported by the Defense Advanced Research Projects Agency (DARPA) under SSC San Diego Contract N66001-04-C-8045.

G. E. Betts is with Photonic Systems, Inc., Billerica, MA 01821 USA (e-mail: gbetts@photonicsinc.com).

X. B. Xie, I. Shubin, W. S. C. Chang, and P. K. L. Yu are with the University of California at San Diego, La Jolla, CA 92093 USA.

Digital Object Identifier 10.1109/LPT.2006.883292

Fig. 1. Small-signal ac equivalent circuit of electroabsorption modulator. The resistor  $R_P$  represents the voltage-dependent modulator photocurrent source.



Fig. 2. Diagram of the link. The link output is the power delivered to  $R_D$ .

it is a voltage-dependent current. The ac voltage on the modulator is  $v_m$ . We will simplify the analysis by setting  $C_M = 0$ . We are looking only at the low-frequency effects of the photocurrent so we can see the gain limit in its simplest form. When  $C_M \neq 0$ , the photocurrent has additional effects such as increasing the 3-dB bandwidth [2], but it does not change the basic effect discussed here.

The optical link is shown in Fig. 2. The link input is the source represented in the equivalent circuit of Fig. 1.

This is a very simple model that considers only two sources of loss: voltage-independent coupling losses ( $t_I$  and  $t_O$ ), and voltage-dependent absorption loss. We ignore distributed scattering losses and voltage-independent absorption. There is no impact of electrorefractive index change (physically this assumes that there is no reflection and that the waveguidesubstrate index difference is much larger than the electrorefractive index change). The optical power in the modulator input waveguide is  $p_{\text{IN}} = p_L t_I$ , where  $p_L$  is the input laser power. The optical power in the modulator output waveguide is  $p_{\text{OUT}}$ . This analysis applies to small signals with ac voltage much less than  $V_{\pi}$ . Therefore, we approximate the electrooptic transfer function by the transmission at the bias point  $t_B$  and the slope at the bias point

$$p_{\rm OUT} = p_{\rm IN} \left( t_B - \frac{\pi}{2V_{\pi e}} v_m \right). \tag{1}$$

#### 1041-1135/\$20.00 © 2006 IEEE

The slope is characterized by  $V_{\pi e}$ , which is the "effective  $V_{\pi}$ "  $V_{\pi e} = (\pi/2)(dT/dV)^{-1}$ , where T represents the electrooptic transfer function (transmission versus voltage) and the derivative is evaluated at the bias point [4].

The photocurrent  $i_P$  is given by  $(p_{\text{IN}} - p_{\text{OUT}})\eta_M$ , where  $\eta_M$  is the modulator responsivity at the bias point, so

$$i_P = p_{\rm IN} \eta_M (1 - t_B) + p_{\rm IN} \eta_M \frac{\pi}{2V_{\pi e}} v_m.$$
 (2)

The coefficient of  $v_m$  has the units of inverse resistance, so we can define the effective small-signal ac photocurrent resistance  $R_P$  as

$$R_P = \frac{2V_{\pi e}}{p_L t_I \eta_M \pi}.$$
(3)

 $(R_P \text{ is contributed to both by the photocurrent generated by absorption, and by reduced junction resistance due to other effects. We represent all these effects by the parameter <math>\eta_M$ .)

The equivalent circuit can be solved to give the modulator voltage  $v_m$  in terms of the source current  $i_s$  as

$$v_m = i_s \frac{R_L R_S}{R_L + R_S} \frac{1}{1 + \frac{p_L t_I \eta_M \pi}{2V_{\pi e}} \left(R_M + \frac{R_L R_S}{R_L + R_S}\right)}$$
(4)

where  $R_L$  is the modulator termination resistance,  $R_S$  is the source impedance,  $R_M$  is the resistance in series with the modulator junction, and the other quantities have been defined earlier. This equation shows the source of the gain limit—the modulator ac voltage is inversely proportional to the optical power at high optical power.

The link gain is the ratio of the output RF power to the input RF power. The input RF power is defined as the power delivered by the source to a matched load, which is the available power  $\langle i_s^2 \rangle R_S/4$ . The link output is the power delivered to the detector load resistance  $R_D$ .

The link gain is given by (5), where  $\eta_D$  is the detector responsivity. We have assumed no losses in the link except the modulator

$$g = \left[ \left( \frac{p_L t_I t_O \eta_D \pi}{2 V_{\pi e}} \right)^2 R_D R_L \right] \left[ \frac{4 R_L R_S}{(R_L + R_S)^2} \right] \\ \times \left[ \frac{1}{1 + \frac{p_L t_I \eta_M \pi}{2 V_{\pi e}} \left( R_M + \frac{R_L R_S}{R_L + R_S} \right)} \right]^2.$$
(5)

The gain is the product of three terms: the link gain for an external modulation link with impedance-matched input, the effect of an impedance mismatch between the source and termination, and a third term with the dependence on the input optical power. In the limit of small  $p_L$ , the third term approaches unity and the link behaves as expected for an external modulation link.

In the limit of large  $p_L$ , the third term becomes inversely proportional to  $p_L$ . In this limit, the gain becomes independent of either  $p_L$  or  $V_{\pi e}$ , and is given by

$$g_{\text{Limit}} = \left(\frac{t_O \eta_D}{\eta_M}\right)^2 \frac{4\frac{R_D}{R_S}}{\left(1 + \frac{R_M}{R_S} + \frac{R_M}{R_L}\right)^2}.$$
 (6)



Fig. 3. Link electrical gain as a function of laser power, for various values of the modulator responsivity  $\eta_M$  (units of  $\eta_M$  are A/W). The dc component of the modulator photocurrent is also plotted. The parameter values are:  $V_{\pi e} = 1$  V,  $R_S = R_L = R_D = 50 \ \Omega$ ,  $R_M = 5 \ \Omega$ ,  $\eta_D = 0.8$  A/W,  $t_I = -2$  dB,  $t_O = -2$  dB, and  $t_B = 0.5$ .



Fig. 4. Modulator input impedance as a function of laser power, for various values of the modulator responsivity. The modulator capacitance is zero in this calculation.  $R_M$  is the modulator series resistance. Parameters are as in Fig. 3.

The source and detector load impedances  $R_S$  and  $R_D$  are constrained by system requirements and are generally equal. The output optical transmission  $t_O$  cannot be larger than one, and the detector responsivity is likewise constrained. The only parameter left to use to increase the gain in this limit is the modulator responsivity [3].

The effect of this gain limit is shown in Fig. 3. The case of  $\eta_M = 0$  is the standard external modulation result with no photocurrent effect. The case with  $\eta_M = 1$  A/W approximates performance expected from a high-power electroabsorption modulator. For a high-performance modulator, the limiting value is near 0 dB. The limit can be increased if the modulator responsivity is reduced, but even at a low responsivity such as 0.1 A/W, the photocurrent effect has an impact.

The input impedance  $Z_M$  of the modulator varies with optical power. The relation is given by

$$Z_M = \frac{1}{\frac{1}{R_L} + \frac{1}{R_M + \frac{2V_{\pi e}}{p_L t_I \eta_M \pi}}}.$$
 (7)

The input impedance is plotted in Fig. 4. While the impedance mismatch does affect the gain, it is not the primary factor in the gain limit.

Authorized licensed use limited to: Gary Betts. Downloaded on August 18, 2009 at 14:53 from IEEE Xplore. Restrictions apply



Fig. 5. Link noise figure. Parameter values are as in Fig. 3.



Fig. 6. Effect of  $V_{\pi e}$  on gain and noise figure. The modulator responsivity is 1 A/W. Other parameters are as in Fig. 3.

The gain limit also results in a minimum noise figure. The link electrical noise figure is given by

$$f = \frac{N_{\text{out}}}{gkT_o} = 1 + \frac{f_R}{g} + \frac{2ep_L t_I t_B t_O \eta_D R_D}{gkT_o} + \frac{ep_L t_I \eta_M \left(1 - t_B\right) R_S \left[1 + \frac{R_M (R_L + R_S)}{R_S R_L}\right]^2}{2kT_o}$$
(8)

where  $N_{\text{out}}$  is the total output noise,  $f_R$  is the receiver noise figure ( $f_R = 1$  in this letter), k is Boltzmann's constant,  $T_o$  is 290 K, and e is the elementary charge. We have assumed there is no relative intensity noise.

The first three terms are the familiar input, receiver, and detector shot noise terms. The fourth term is due to shot noise from the dc component of the modulator photocurrent. For small  $\eta_M$  or for low bias (small  $t_B$ ) the modulator shot noise term becomes the dominant term at high optical power. The noise figure is plotted in Fig. 5.

Even though the gain is independent of  $V_{\pi e}$ , there is some advantage to a low  $V_{\pi e}$ . The gain reaches its limit at a lower optical power. The minimum noise figure is lower and it occurs at a lower optical power. Fig. 6 shows these effects.

#### **III. EXPERIMENT**

We have verified the gain limit by measuring the gain of a link using an electroabsorption modulator at high optical power levels. The modulator structure is similar to that described in [5]. The  $V_{\pi e}$  was 0.85 V and the input and output losses were ap-



Fig. 7. Experimental measurement of a link using an electroabsorption modulator at 1550 nm, compared with the theoretical gain calculation.

proximately  $t_I = t_O = 0.5$ . The bias point was  $t_B = 0.5$ , which occurred at 1.5-V reverse bias. The ac input voltage was 0.063-V peak-to-peak. The modulator's apparent dc responsivity varied from 0.7 to 1.5 A/W, indicating some mechanism creating additional photocurrent beyond simple absorption. An RF responsivity  $\eta_M = 0.8$  A/W was used to fit the calculation to the measured data. The measurement frequency was 50 MHz, well below the *RC* bandwidth.

The results are shown in Fig. 7. The gain follows the theoretical prediction very closely. The gain deviates from the prediction of this model only at the highest powers used (>250 mW) due to heating.

## IV. CONCLUSION

There is a limit on the performance of external modulation analog links using electroabsorption modulators at high optical power. The link gain is limited to a value dependent on the modulator's responsivity and a few other parameters; the gain limit is independent of the optical power or the effective  $V_{\pi}$ . The link noise figure also has a minimum value which large optical power cannot improve. The link gain limit has been verified experimentally.

## ACKNOWLEDGMENT

The authors would like to acknowledge S. Pappert for helpful discussions of the gain limit.

#### REFERENCES

- C. H. Cox III, Analog Optical Links: Theory and Practice. Cambridge, U.K.: Cambridge Univ. Press, 2004, ch. 3.
- [2] G. L. Li, C. K. Sun, S. A. Pappert, W. X. Chen, and P. K. L. Yu, "Ultrahigh-speed traveling-wave electroabsorption modulator—Design and analysis," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 7, pt. 2, pp. 1177–1183, Jul. 1999.
- [3] L. A. Johansson, Y. A. Akulova, G. A. Fish, and L. A. Coldren, "High optical power electroabsorption waveguide modulator," *Electron. Lett.*, vol. 39, pp. 364–365, Feb. 20, 2003.
- [4] W. S. C. Chang, Ed., RF Photonic Technology in Optical Fiber Links. Cambridge, U.K.: Cambridge Univ. Press, 2002, ch. 6.
- [5] J. X. Chen, Y. Wu, W. X. Chen, I. S. Shubin, A. C. Clawson, W. S. C. Chang, and P. K. L. Yu, "High-power intrastep quantum well electroabsorption modulator using single-sided large optical cavity waveguide," *IEEE Photon. Technol. Lett.*, vol. 16, no. 2, pp. 440–442, Feb. 2004.