The "Effective" Gains and Noise Figures of Individual Components in an Analog Photonic Link

Edward I. Ackerman

Photonic Systems, Inc.
103 Terrace Hall Ave., Suite A
Burlington, MA 01803
eackerman@photonicsinc.com

Abstract—This paper defines "effective" gains and noise figures for each component in an analog photonic link which, when inserted into the equation for the noise figure of cascaded components, yield the established link noise figure equation. Whereas the square-law electro-optic devices (modulator and photodetector) have effective gains that are—not surprisingly—dependent on RF power levels, it is found that the effective noise figures of the components that dominate the link noise figure are independent of RF power over the link's range of useful RF input power levels.

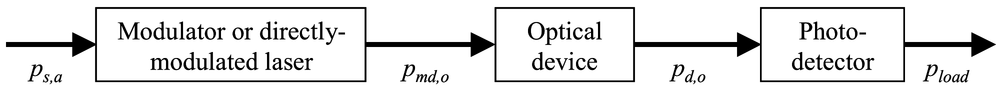
I. Background

To enable analysis of the performance of a system in which an analog optical link routes an RF signal between two points, expressions for the RF gain (or, more often, RF loss) and noise figure of a number of different link configurations have been derived and experimentally verified (see, for example, [1]). Although an analog link necessarily consists of at least three components—the optical source and modulator (which is a single semiconductor laser in a direct modulation link, or two separate components in an external modulation link), the optical propagation medium, and the photodetector, all past investigations have derived the performance of the RF-in, RF-out "black box" comprising the complete link rather than attempting to express the gains and noise figures of the individual components.

In this paper, "effective" small-signal gains and noise figures are derived for the individual components. Because the optical modulation and detection processes are square-law in nature, the effective gains of the modulator (or directly modulated laser) and photodetector have an RF input or output power dependency and are therefore of limited usefulness. Deriving the effective noise figures of the modulator, optical medium, and photodetector, however, proves very useful because it illustrates what technology developments are most critical to enabling higher-performance links, and does so in a fashion that is more meaningful to RF system designers than previous full-link analyses could.

II. Theory

Figure 1 shows how an early link modeling investigation by Cox *et al.* [2] defined the individual small-signal efficiencies of the three fundamental components in an intensity-modulation direct-detection link. Because an external modulator or directly modulated laser imposes signal sidebands on an optical carrier that have a power, $p_{m,o}$ or $p_{l,o}$, proportional to the square root of the available input RF source power $p_{s,a}$, and because the RF power delivered by the photodetector to its output load impedance, p_{load}, is proportional to the square of the power in these sidebands, their definitions of incremental modulation and detection efficiencies were independent of input RF power levels over the entire range for which the signal can be said to be small (i.e., whenever the modulation index is << 1).



Definitions of Cox *et al.* [2] :

Incremental modulation efficiency $\equiv \dfrac{p_{md,o}^2}{p_{s,a}}$ Loss$^2 \equiv \dfrac{p_{d,o}^2}{p_{md,o}^2}$ Incremental detection efficiency $\equiv \dfrac{p_{load}}{p_{d,o}^2}$

This paper :

Effective gain of modulation device $\equiv \dfrac{p_{md,o}}{p_{s,a}}$ Loss $\equiv \dfrac{p_{d,o}}{p_{md,o}}$ Effective gain of detector $\equiv \dfrac{p_{load}}{p_{d,o}}$

Fig. 1 Analog optical link component efficiencies (as defined by Cox et al. [2]) and effective gains (as defined in this paper).

To determine "effective" noise figures for the components in Figure 1 requires that effective gains be defined differently than the incremental modulation and detection efficiencies derived by Cox *et al*. [2]. For example, to be useful to a system designer the effective noise figures need to satisfy the following well-established equations for the gain and noise figure of a cascade of three components:

$$g_{link} = g_{md}\, g_{od}\, g_d \ , \tag{1}$$

$$nf_{link} = nf_{md} + \frac{nf_{od} - 1}{g_{md}} + \frac{nf_d - 1}{g_{md}\, g_{od}} \ . \tag{2}$$

In these equations g_{md}, g_{od}, and g_d will be called, for lack of better terms, the effective gains of the modulation device (i.e., an external modulator or a directly modulated laser), the optical device, and the photodetector. The corresponding *nf* terms will be called their effective noise figures. Equation (2) requires that the *nf* and *g* terms all be dimensionless.

Ignoring for the moment the square-law nature of the modulation and detection processes, the effective gain and noise figure can be defined for any device at a specific input signal power:

$$g_{device} \equiv \frac{p_{output}}{p_{input}} \ , \tag{3}$$

$$nf_{device} \equiv \frac{noise\ power_{output}}{k\,T\,B \times g_{device}} . \tag{4}$$

These definitions can be applied to yield appropriate expressions for each component. The result of such derivations is shown here for links with perfect, lossless input and output impedance matching circuits.

Directly modulated laser—

$$g_l = \frac{s_l\, N_L}{\sqrt{R_S\, p_{s,a}}} = \frac{s_l^2\, N_L^2}{R_S\, p_{l,o}} \ , \tag{5}$$

$$nf_l = 2 + \frac{I_L^2\, RIN\, R_S}{k\,T\, N_L^2} + \frac{1}{g_l} . \tag{6}$$

Mach-Zehnder external modulator—

$$g_m = \frac{\pi P_I\, t_{ff}\, N_M}{2\,V_\pi} \sqrt{\frac{R_S}{p_{s,a}}} = \left(\frac{\pi P_I\, t_{ff}}{2\,V_\pi}\right)^2 \frac{N_M^2\, R_S}{p_{m,o}} \ , \tag{7}$$

$$nf_m = 2 + \frac{V_\pi^2\, RIN}{k\,T\, N_M^2 R_S} + \frac{1}{g_m} . \tag{8}$$

Passive optical device—

$$g_{od} = t_{od} \ , \tag{9}$$

$$nf_{od} = \frac{1}{g_{od}} = \frac{1}{t_{od}} . \tag{10}$$

Photodetector—

$$g_d = r_d^2\, N_D^2\, R_{LOAD}\, p_{d,o} \ , \tag{11}$$

$$nf_d = \frac{2\,q\, I_D\, N_D^2\, R_{LOAD}}{k\,T\, g_d} + \frac{1}{g_d} . \tag{12}$$

In equations (5) – (12), k is Boltzmann's constant, T = 290 K, R_S and R_{LOAD} are the RF source and load impedances presented to the link, the squares of N_L, N_M, and N_D quantify the gain improvements obtained by using lossless matching circuits to transform the laser, modulator and detector impedances to the source and load impedances, s_l is the laser's slope efficiency, I_L is its above-threshold bias current, *RIN* is its relative intensity noise, P_I is the optical power into the external modulator, t_{ff} and V_π are the Mach-Zehnder modulator's fiber-to-fiber insertion loss and on-off switching voltage, r_d is the photodetector's responsivity, and I_D is its dc photocurrent.

Substituting equations (9) and (11) along with either (5) or (7) into (1) yields the well-established expressions for the small-signal gains of, respectively, either a direct or external modulation link with perfect, lossless, passive input and output impedance matching circuits [2]. Similarly, substituting (10) and (12) along with either (6) or (8) into (2) yields expressions for the noise figure of such links that were also derived and experimentally verified previously [3]. (*Note*: the nf_m expression for a Mach-Zehnder modulator with a different input match condition [3] or with a traveling-wave electrode [4] would likely have an additive constant other than 2.)

III. Discussion

Figure 2(a) shows how the individual components' gains contribute to the total link gain across a –60 to 0 dBm range of available RF source powers. These curves were calculated using equations (3), (7), (9), and (11) assuming an external modulation link having a Mach-Zehnder modulator, optical device, and photodetector with the parameters listed in the figure caption. The range of source powers shown was chosen somewhat arbitrarily; in actuality, an RF power of 0 dBm would qualify as a large-signal input for this modulator. Ignoring that fact for the moment, notice that for any value of $p_{s,a}$ the effective gains (in dB) of the three individual components add up to that of the complete link, which is approximately 0 dB in this case.

Figure 2(b) shows the effective noise figures for these same components. The detector's effective noise figure decreases with increasing $p_{s,a}$; the effective noise figure of the optical device is independent of $p_{s,a}$ and, interestingly, so is that of the modulator over this range of $p_{s,a}$. If the plot were extended to larger values of $p_{s,a}$, it would show nf_m beginning to increase. However, 0 dBm already qualifies as quite a large signal for this modulator, so nf_m can be considered to be constant over the range of interesting $p_{s,a}$.

Figure 2(c) is more useful than 2(b) because instead of showing the effective noise figures of the components it shows how equation (2) dictates their contributions to the total noise figure. Notice that for these component values the optical device's 10 dB loss contributes only negligibly to noise figure. The modulator and detector contributions dominate, and are flat over this range of RF powers. This is interesting and unexpected given that the gains of both devices depend on input RF power.

Because the important contributors to the link noise figure are independent of RF power level, the data from Figure 2 can be plotted vs. the optical component's effective gain (or in the case of this passive optical component, its loss) for a single value of $p_{s,a}$ (for example, 0 dBm). The gain of the link is proportional to the square of the optical component's effective gain, as shown in Figure 3(a). Figure 3(b) shows the effective noise figures of the three components; an arrow is shown on the nf_d curve to indicate how it translates upwards by 0.5 dB for each 1 dB decrease in $p_{s,a}$. However, again it is more illustrative to examine Figure 3(c), which shows how the addends in equation (2) depend on g_{od}. The two dominant addends are independent of $p_{s,a}$, and the remaining addend is negligible when $p_{s,a}$ = 0 dBm (which, again, is beyond the small-signal limit) and only gets smaller as $p_{s,a}$ is decreased.

Figure 3(c) shows clearly now what link designers have long known: the modulator characteristics set the limit on the attainable link noise figure (even when V_π = 0.3 V, as was assumed for Figures 2 and 3), except in cases where the optical loss is quite large.

IV. Conclusions

Derivation of the "effective" gains and noise figures of the individual components in an intensity-modulation direct-detection RF optical link enables an examination of the cascade of components that comprise the link in the manner to which RF designers are accustomed. Such an analysis shows clearly that even for an external modulation link with a very high-efficiency modulator (e.g., V_π = 0.3 V), the modulator's effective noise figure dominates that of the complete link.

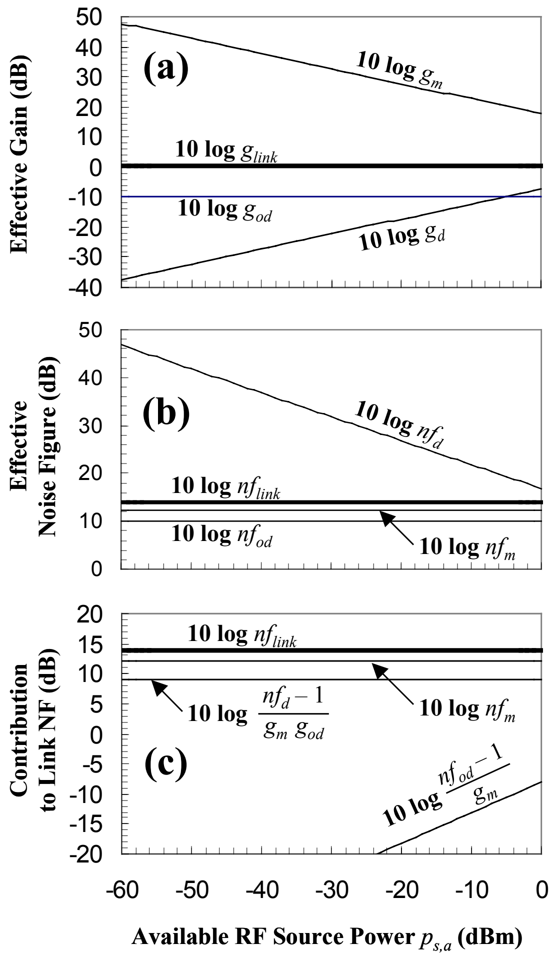


Fig. 2 Plots vs. the available RF source power of (a) effective gains, (b) effective noise figures, and (c) addends in equation (2), assuming a Mach-Zehnder modulator-based external modulation link with 10 dB of optical loss (g_{od} = 0.1) and the following characteristics: $R_S = R_{LOAD}$ = 50 Ω; $N_M = N_D$ = 1; RIN = −155 dB/Hz; P_I = 100 mW; t_{ff} = 0.5; V_π = 0.3 V; r_d = 0.8 A/W.

References

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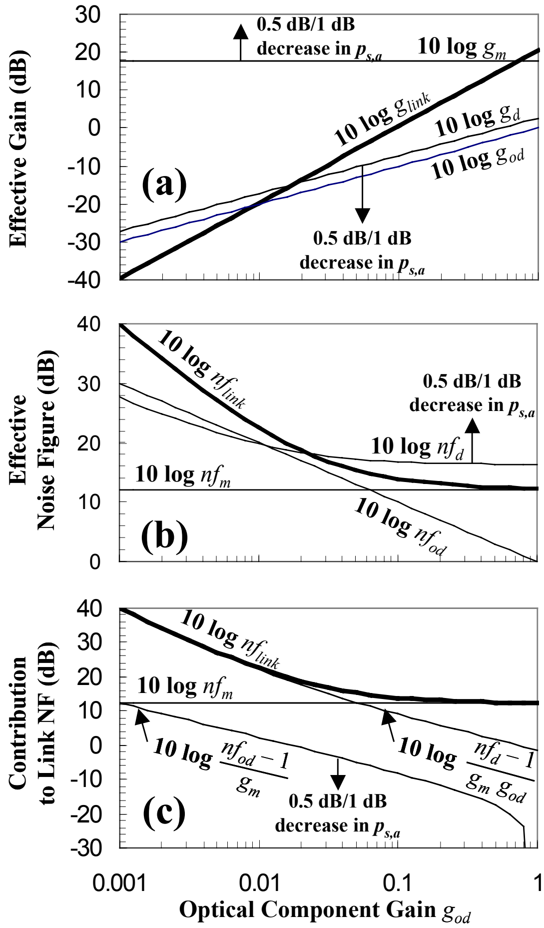

Fig. 3 Plots vs. the optical component gain, g_{od}, of (a) effective gains, (b) effective noise figures, and (c) addends in equation (2), assuming the same components as in Fig. 2 (except that g_{od} is now a variable), for $p_{s,a}$ = 0 dBm.